Microeconomic Theory
Lecture 3: Household Models

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1. Motivation

Most Third World producers are self-employed, as farmers or informal sector/micro-entrepreneurs. We therefore need a model that captures their behavior. An additional justification is that household models are an excellent introduction to subsequent models (general equilibrium, risk aversion, saving and consumption, growth).

We first cover unitary household models, that is, models that assume away dissention within the household as to whose preferences ought to be represented in the household's choices.

2. Household model without missing markets

Readings: Singh, Squire and Strauss, Agricultural Household Models, The World Bank, Washington DC, 1986 – Chapters 2 and 3.

2.1. The basic model (as in Singh, Squire and Strauss)

The basic household model is a combination of consumer and producer models into a single model, i.e., the consumer is also the entrepreneur. We take the example of hypothetical farming household. The household has preferences on, say, an agricultural good X_a, a manufactured good X_m, and leisure X_l defined by the utility function $U(X_a, X_m, X_l)$. The household has a production technology represented by the production function $Q(L, A)$ where L is labor and A is land (acreage). Land is assumed fixed (short-run choices). The budget function of the household is:

$$p_m X_m = p_a(Q(L, \bar{A}) - X_a) + w(F - L)$$

where w is the wage rate and p_a and p_m are market prices. On the left hand side we have expenditures on manufactures. These expenditures are financed by sales of agricultural products (production minus consumption, or marketed surplus, $Q(L, \bar{A}) - X_a$) and net sales of labor on the labor market $F - L$. In addition to the budget constraint, the household must also satisfy a time accounting identity:

$$X_l + F = \bar{T}$$

where $\bar{T}$ is total time, which is fixed, and F is total family labor.

Utility maximization can be written:

$$\max_{X_a, X_m, X_l, L} U(X_a, X_m, X_l)$$

subject to the so-called full income budget constraint:

$$p_m X_m + p_a X_a + w X_l = w\bar{T} + p_a Q(L, \bar{A}) - wL$$

The left hand side of the constraint represents the value of all consumption, including own produced food and leisure. The right hand side represents the so-called full income. It is made of $w\bar{T}$, the value of total time evaluated at the market wage rate, and profits $p_a Q - wL$ (revenues minus labor costs). The full income budget constraint is obtained by replacing F using the time accounting identity.

The first order condition for optimization with respect to labor L is:

$$\lambda \left[\frac{\partial Q}{\partial L} p_a - w \right] = 0$$

where λ is the Lagrange multiplier. Since λ is always positive (more money is better, hence the budget constraint is always constraining), it can be factored out of the first order condition for labor. This implies that labor choices do not depend on λ. We are left with an expression identical to the one we would have gotten if the decision maker was a profit maximizer. Solving the above first order condition yields an optimal value of labor $L^*(p_a, w, \bar{A})$ as well as an optimal value of output $Q^*(p_a, w, \bar{A})$. If we insert it into the definition of profits, we get the profit function $\pi(p_a, w, \bar{A})$ which, combined with $w\bar{T}$, fully determines full income Y^*.

2.2. First order conditions and recursivity

The first order conditions for consumption goods are of the form:

$$\frac{\partial U}{\partial X_i} = \lambda p_i$$

for $i = a, m, l$, together with the full income budget constraint:

$$\sum_i p_i X_i = Y^*$$

It is easy to see that the above system of first order conditions is the same as the one we got in the consumer utility maximization problem, with Y^* replacing m. Solving the system of first order conditions yields an indirect utility function as well as the usual demand system of the form $X_i(p_a, p_m, w, Y^*)$.

The property that production decisions in such household models are independent of (i.e., can be determined without knowing) household consumption preferences is called *separability* although, strictly speaking, it should be termed recursivity. Indeed, although production choices do not depend on consumption preferences, consumption choices depend on production choices through Y^*.

2.3. Marketed surplus and supply response

The above household model provides a useful benchmark to study the response of market surplus $M \equiv Q - X_a$ to changes in price p_a. We begin by noting that:

$$\frac{dX_a}{dp_a} = \frac{\partial X_a}{\partial p_a} + \frac{\partial X_a}{\partial Y^*} \frac{\partial Y^*}{\partial p_a}$$

The response of consumption to a change in the price of agricultural products has two components: a standard Marshallian price effect (first term), and an income effect (second term). The income effect comes from the fact that agricultural products are one of the source of household income. Now, since $Y^* = wT + \pi(p_a, w, \bar{A})$, we have:

$$\frac{\partial Y^*}{\partial p_a} = \frac{\partial \pi(p_a, w, \bar{A})}{\partial p_a} = Q^*$$

by Hotelling's lemma. Putting everything together, we have:

$$\frac{\partial M}{\partial p_a} = \frac{\partial Q^*}{\partial p_a} - \frac{\partial X_a}{\partial p_a} - \frac{\partial X_a}{\partial Y^*} Q^*$$

We know that the first term is necessary positive (why?). The second term is also normally positive (negative with negative sign in front). But the last term is normally negative (positive with negative sign in front). Consequently, the sign of the above expression is, in general, ambiguous: marketed surplus might decrease when output price increases. This will occur when the last term is large, i.e., when the income elasticity of agricultural consumption is high. To get a better sense of the anticipated magnitude of the marketed surplus response, let us rewrite the above equation in elasticity terms:

$$\varepsilon_M = \frac{\varepsilon_a}{1 - s_a} - \frac{\theta_a s_a}{1 - s_a} - \frac{\eta_a s_a r_a}{1 - s_a}$$

where ε_a is the output price elasticity, θ_a is the consumption price elasticity, η_a is the (full) income elasticity of consumption, s_a is the share of self-consumption in output $\frac{X_a}{Q}$, and r_a is the share of agricultural revenue in full income $\frac{p_a Q}{Y^*}$. With these definitions, the above equation can serve to compute the market surplus elasticity.

3. Household model with missing markets

Readings: de Janvry, Fafchamps and Sadoulet, "Peasant household behavior with missing markets: some paradoxes explained", Economic Journal, 101 (409): 1400-17, November 1991

3.1. The model

In practice, farming households in poor countries often face missing or incomplete markets, due to geographical isolation and imperfect institutions. In the presence of missing markets, households' response to market changes are likely to be quite different. The household model provides a useful framework in which to study these responses. We begin with a slightly more general notation and compact notation for the household model, borrowing heavily from de Janvry, Fafchamps, and Sadoulet (1991). The household is assume to solve the following optimization problem:

$$\max_{\{c,q\}} U(c)$$

subject the monetary budget constraint:

$$\sum_{i \in T} p_i c_i = \sum_{i \in T} p_i (q_i + T_i) + S$$

where p_i is the price of good i, c_i is the quantity consumed of good i, q_i is the quantity of good i in production (output if positive, input if negative), T_i is the household's initial endowment of good i (e.g., time), and S is an unearned supplementary cash income. The set of traded goods is denoted T. The household has at its disposal a production function:

$$G(q) = 0$$

and a self-sufficiency constraint for all non-traded goods:

$$q_i + T_i = c_i \text{ for all } i \in N$$

For notation purposes, we also note that market prices are fixed for the household:

$$p_i = \bar{p}_i \text{ for all } i \in T$$

This optimization problem can be rewritten as the Lagrangian:

$$£ : U(c) + \lambda \left[\sum_{i \in T} p_i(q_i + T_i - c_i) + S \right] + \phi G(q) + \sum_{i \in N} \mu_i (q_i + T_i - c_i)$$

First order conditions with respect to consumption goods are of the form:

$$\begin{aligned} U_i' &= \lambda p_i \text{ for all } i \in T \\ U_i' &= \mu_i \text{ for all } i \in N \end{aligned}$$

First order conditions with respect to production outputs and inputs are of the form:

$$\begin{aligned} G_i' &= -\frac{\lambda}{\phi} p_i \text{ for all } i \in T \\ G_i' &= -\frac{\mu_i}{\phi} \text{ for all } i \in N \end{aligned}$$

To simplify the notation, we now use a notational trick and define shadow prices for non-traded goods as:

$$p_i \equiv \frac{\mu_i}{\lambda} \text{ for all } i \in N$$

With notational convention, the first order conditions can be written as the following joint system of equations:

$$U_i' = \lambda p_i \text{ for all goods consumed} \tag{3.1}$$

$$G_i' = -\frac{\lambda}{\phi} p_i \text{ for all goods produced} \tag{3.2}$$

$$\sum_{i \in N,T} p_i c_i = \sum_{i \in N,T} p_i (q_i + T_i) + S \tag{3.3}$$

$$G(q) = 0 \tag{3.4}$$

$$q_i + T_i = c_i \text{ for all } i \in N \tag{3.5}$$

$$p_i = \bar{p}_i \text{ for all } i \in T \tag{3.6}$$

The first set of equations are the first order conditions for utility maximization; the second set of equations are profit maximization conditions; the third equation is the full income budget constraint; the fourth one is the production function (in implicit form); the fifth one is the self-sufficiency condition for non-traded goods; and the last one is the condition that prices equal market prices for traded goods. The second set of equations, together with the production function, can be used to compute the profit function $\pi(p)$. Given $\pi(p)$, full income is obtained as:

$$Y = \pi(p) + S + \sum_{i \in N,T} p_i T_i$$

Plugging the above into the budget constraint and combining with the first set of equations, we can compute the indirect utility function $V(Y,p)$. The fifth and sixth set of equations nail down the prices. The above system can thus be rewritten as a system of input demand/output supply equations, consumption demand equations, and 'market clearing' conditions:

$$q_i = q_i(p) \tag{3.7}$$

$$c_i = c_i(Y,p) \tag{3.8}$$

$$Y = \pi(p) + S + \sum_{i \in N,T} p_i T_i \tag{3.9}$$

$$q_i + T_i = c_i \text{ for all } i \in N \tag{3.10}$$

$$p_i = \bar{p}_i \text{ for all } i \in T \tag{3.11}$$

So doing, we have formally turned our household model into a little partially closed economy. The price of traded goods is determined by the market price; the shadow price of non-traded goods is determined by the self-sufficiency condition. It is as if the household had a multiple personality disorder: the producer side of the household maximizes profits, given market and shadow prices, while the consumer side of the household maximizes utility given market and shadow prices. Equations 9, 10 and 11 ensure that producer and consumer pick production and consumption plans that are consistent, i.e., that quantities produced plus initial endowments equal consumption (in case of non-traded goods), and that monies spent on the consumption of traded goods is generated by the sale of outputs or endowments.

3.2. Market response

The response of households with missing markets to changes in market conditions can be extremely different from that of households facing markets for all goods and factors of production. To see why, let us consider a special case with a single missing market and let us analyze what is the household's response to a change in price of an unconsumed output which, to facilitate the discussion, we call a cash crop. Totally differentiating equation 3.7 we get:

$$\frac{dq_i}{dp_i} = \frac{\partial q_i(p)}{\partial p_i} + \sum_{j \in N} \frac{\partial q_i}{\partial p_j} \frac{dp_j}{dp_i} \tag{3.12}$$

The above equation states that the effect of a change in the price of the cash crop is the sum of two effects: a direct output response and an indirect output response due to the possible effect of the change in market price on shadow prices internal to the household. To make sense of equation 3.12, we need to know how shadow prices change with market price p_i. This can be found by totally differentiating 3.10 (see de Janvry, Fafchamps and Sadoulet, 1991, for details).

Simulation exercises show that the response to a change in cash crop prices is likely to be smaller when the household is constrained to be self-sufficient in food and/or labor; it can even be perverse (why?).

4. Empirical work

There is an enormous body of empirical work that uses household models as starting point. In fact, virtually all microeconomic work on rural households uses this modeling framework in one way or another. Here are some examples:

- Test of separability of the household model (e.g., Benjamin, Econometrica, 1992).
- Estimation of separable household models to compute marketed surplus supply response (e.g., Singh, Squire and Strauss, 1986).
- Estimation of (parts of) non-separable household models: output supply; input demand; consumption demand; labor supply; including consumption of welfare type goods such as education, health, child nutrition, micro-nutrients, etc.

In many cases reliance on household models is only implicit, as when researchers estimate a production function using village-level data: only if certain markets are incomplete/imperfect is the estimation possible since, if all producers faced the same factor prices and had the same production function, they would use the same level of inputs and estimation would be impossible.